Teaching Student-Centered MATHEMATICS Grades 3-5

The Workbook

John A. Van de Walle LouAnn H. Lovin

Chapter 2 Developing Early Number Concepts and Number Sense

	Activity	Mathematical Goal	page number (workbook/text)	
2.1	Little-Ten Frames	Extending number relationships to larger numbers	1	43
2.2	Who Could They Be?	Develop relative magnitude of numbers to 100	4	45
2.3	Is It Reasonable?	Develop relative magnitude of numbers to 100	6	46
2.4	What Comes Next?	Develop the continuing pattern in the place-value system	7	48
2.5	The Other Part of 100	Developing missing-part strategies with a whole of 100	8	54
2.6	Compatible Pairs	Explore combinations that make multiples of 10 or 100	11	55
2.7	Calculator Challenge Counting	Develop mental addition strategies through skip counting	12	55
2.8	Little Ten-Frame Addition and Subtraction	Develop invented strategies for addition and subtraction	13	55
2.9	Interpret the Remainder	When dividing, develop an understanding of how to interpret the remainder	16	61
2.10	Finding Factors	Develop connection between multiplication and division	18	63
2.11	Factor Patterns	Connect factors with rectangular arrays	19	63
2.12	Learning About Division	Develop measurement and partition concepts of division	22	64
2.13	Slice It Up	Develop the distributive property	24	66

Chapter 3 Helping Children Master the Basic Facts

	Activity	Mathematical Goal	page number (workbook/text)	
3.1	Clock Facts	Develop minutes after on the clock as a strategy for fives multiplication facts	26	88
3.2	Patterns in the Nines Facts	Develop strategy for nines multiplication facts	27	90
3.3	If You Didn't Know	Open-ended task for inventing strategies for basic facts	28	92
3.4	How Close Can You Get?	Practice finding missing factors in a division relationship	30	93

Chapter 4 Strategies for Whole-Number Computation

	Activity	Mathematical Goal	page number (workbook/text)	
4.1	Ten-Frame Adding and Subtracting	Connect basic-fact strategies involving ten- to two-digit numbers	31	108
4.2	Cluster Problems	Develop a conceptual understanding of how to perform multiplication with two-digit multipliers	35	117
4.3	Invented Strategies for Multiplication	Develop a conceptual understanding of how to perform multiplication with one-digit multipliers	37	119

Chapter 5 Developing Fraction Concepts

	Activity	Mathematical Goal	page number (workbook/text)	
5.1	Word Problems	Develop an understanding of how to form fair shares	41	132
5.2	Correct Shares	Develop an understanding of fractional parts	43	136
5.3	Finding Fair Shares	Develop an understanding of fractional parts	44	136
5.4	More, Less, or Equal to One Whole	Develop an understanding of fractional parts	45	138
5.5	Mixed-Number Names	Develop an understanding of mixed numbers	47	140
5.6	Find the Part	Parts-and-whole tasks	49	141
5.7	Find the Whole	Parts-and-whole tasks	50	142
5.8	Find the Fraction	Parts-and-whole tasks	51	142
5.9	Zero, One-Half, or One	Develop benchmarks of 0, 1/2, and 1 for fractions	52	145
5.10	Close Fractions	Develop benchmarks of 0, 1/2, and 1 for fractions	54	145
5.11	About How Much?	Develop an understanding of the size of fractions	57	145
5.12	Ordering Unit Fractions	Develop an understanding of the size of fractions	59	146
5.13	Choose, Explain, Test	Develop an understanding of the relative size of fractions	60	148
5.14	Line ‘Em Up	Develop an understanding of the relative size of fractions	63	148
5.15	First Estimates	Develop an understanding of the relative size of fractions through estimation	66	150
5.16	Different Fillers	Develop an understanding of the concept of equivalent fractions	68	152
5.17	Dot Paper Equivalencies	Develop an understanding of the concept of equivalent fractions	70	152
5.18	Group the Counters, Find the Names	Develop an understanding of the concept of equivalent fractions	72	153
5.19	Missing-Number Equivalencies	Use equivalent fraction concepts to find specific equivalent fractions	74	155
5.20	Slicing Squares	Develop an algorithm for equivalent fractions	76	155

Chapter 6 Fraction Computation

	Activity	Mathematical Goal	page number (workbook/text)	
6.1	LCM Flash Cards	Practice finding the lease common multiple of a pair of numbers	78	166

Chapter 7 Decimal and Percent Concepts and Decimal Computation

	Activity	Mathematical Goal	page number (workbook/text)	
7.1	Expanded Form	Develop a conceptual connection between fractions and decimal notations	79	183
7.2	Base-Ten Fractions to Decimals	Develop a conceptual connection between fractions and decimal notations	80	186
7.3	Calculator Decimal Counting	Develop an understanding of the patterns in decimal notation	82	187
7.4	Friendly Fractions to Decimals	Develop a conceptual connection between fractions and decimal notations	83	188
7.5	Estimate, Then Verify	Develop a conceptual connection between fractions and decimal notations	84	188
7.6	Decimals on a Friendly Fraction Line	Develop a conceptual connection between fractions and decimal notations	86	189
7.7	Close to a Friendly Fraction	Practice Estimation of decimal numbers with simple fractions	87	190
7.8	Best Match	Practice Estimation of decimal numbers with simple fractions	89	191
7.9	Line 'Em Up	Develop an understanding of the way that decimal numbers are ordered	90	191
7.10	Close "Nice" Numbers	Develop an understanding of the relative size of decimal numbers	92	192
7.11	Exact Sums and Differences	Develop invented strategies for decimal addition and subtraction	93	198
7.12	Where Does the Decimal Go? Multiplication	Use estimation to place the decimal point in multiplication	95	199
7.13	Where Does the Decimal Go? Division	Use estimation to place the decimal point in division	96	200

Chapter 8 Geometric Thinking and Geometric Concepts

	Activity	Mathematical Goal	page number (workbook/text)	
8.1	Shape Sorts	Develop ways that 2-D shapes are alike and different	97	212
8.2	What's My Shape?	Develop oral descriptions of shapes	99	213
8.3	Geoboard Expansion	Develop spatial sense with 2-D shapes	107	215
8.4	Congruent Parts	Develop spatial sense with 2-D shapes	109	216
8.5	Can You Make It?	Develop awareness of properties of shapes	110	217
8.6	Mystery Definition	Develop defining properties of special classes of shapes	111	224
8.7	Triangle Sort	Develop standard classifications of triangles by sides and angles	112	225
8.8	Property Lists for Quadrilaterals	Explore all properties attributable to special classes of quadrilaterals	115	226
8.9	Diagonal Strips	Explore the relationships between diagonals for different types of quadrilaterals	120	227

8.10	Minimal Defining Lists	Explore alternative definitions of classes of quadrilaterals	123	230
8.11	True or False	Explore informal deductive statements concerning properties of shapes	124	231
8.12	Motion Man	Develop the concepts of slides, flips, and turns	126	233
8.13	Pattern Block Mirror Symmetry	Develop the concept of line symmetry	129	234
8.14	Dot Grid Line Symmetry	Develop the concept of line symmetry	130	235
8.15	Plane Symmetry Buildings	Extend the concept of line symmetry to three dimensions	133	235
8.16	Plane Block Rotational Symmetry	Develop the concept of rotational symmetry (point symmetry)	134	236
8.17	Coordinate Slides	Develop the way coordinates are used to control translations	135	240
8.18	Coordinate Reflections	Explore the effect on coordinates when shapes are reflected about an axis	138	240
8.19	Coordinate Dilations	Develop the way coordinates are used to control dilations	141	241
8.20	Petominoes	Develop early spatial visualization skills	143	243
8.21	Notches and Holes	Develop spatial visualization skills	145	244
8.22	Viewpoints	Develop spatial visualization skills	146	245
8.23	Perspective Drawings	Develop spatial visualization skills	149	246

Chapter 9 Develop Measurement Concepts

	Activity	**Mathematical Goal**	**page number (workbook/text)**	
9.1	Changing Units	Explore the inverse relationship between unit size and measure	152	258
9.2	Rectangle Comparison – No Units	Develop the concept of area	154	261
9.3	Tangram Areas	Develop the concept of area	156	262
9.4	Fill and Compare	Develop an understanding of units to measure area	159	263
9.5	Rectangular Comparison – Square Units	Develop an understanding of units to measure area; readiness for rectangle area formula	161	264
9.6	Fixed Perimeters	Explore the relationship between area and perimeter of rectangles when the perimeter is constant	163	265
9.7	Fixed Areas	Explore the relationship between area and perimeter of rectangles when the area is constant	166	265
9.8	Capacity Lineup	Develop the concept of capacity	169	266
9.9	Box Comparison – Cubic Units	Develop the concept of volume; readiness for volume formula for prisms	160	267
9.10	One-Handed Clocks	Develop an understanding of the hour hand in reading a clock	171	270

9.11	A Unit Angle	Develop an understanding of the hour hand in reading a clock	173	272
9.12	About One Unit	Develop familiarity with standard units (any attribute)	174	275
9.13	Familiar References	Develop real-world benchmarks or references for standard units (any attribute)	176	276
9.14	Personal Benchmarks	Develop benchmarks of length using body lengths (fingers, hands, etc.)	178	276
9.15	Guess the Unit	Connect measurement to real-world referents	179	277
9.16	Estimation Quickie	Practice measurement estimation (any attribute)	180	280
9.17	Estimation Scavenger Hunt	Practice measurement estimation in real contexts (any attribute)	181	280
9.18	E-M-E Sequences	Develop measurement estimation skills (any attribute)	182	280

Chapter 10 Algebraic Reasoning

	Activity	**Mathematical Goal**	**page number (workbook/text)**	
10.1	Predict Down the Line	Explore the structure of repeating patterns analytically	184	291
10.2	Grid Patterns	Explore how repeat patterns create new patterns when modeled in an array	187	292
10.3	Extend and Explain	Explore growing patterns	191	294
10.4	Predict How Many	Develop functional relationships in growing patterns	194	294
10.5	Find the Function in the Pattern	Develop functional relationships in growing patterns	197	297
10.6	What's Next and Why?	Explore relationships in number patterns	201	299
10.7	Start and Jump Numbers	Develop the ability to analyze numeric patterns	202	299
10.8	One Up and One Down, Multiplication	Develop an understanding of the effect of complementary changes in two factors of a product	204	301
10.9	Diagonal Sums	Explore patterns in the hundreds chart based on addition	206	302
10.10	Story Translations	Representing contextual situations with symbolic expressions	208	307
10.11	Number Tricks	Develop a concept of variable as an unknown quantity	209	308
10.12	What's True for All Numbers?	Develop a concept of variable as a pattern generator	210	308
10.13	Special Quantities	Develop a concept of variable as a pattern generator	211	309
10.14	Tilt or Balance	Develop an understanding of the equal sign and the less-than and greater-than symbols	212	311
10.15	Real-World Functions	Explore the concept of function within real-world situations	214	315
10.16	Guess My Rule	Develop ability to discover functional relationships or operation rules	215	316

Chapter 11 Exploring Data Analysis

	Activity	Mathematical Goal	page number (workbook/text)	
11.1	What's the Meaning of This?	Initial exploration of concepts of mean, median, and mode	217	324
11.2	Leveling the Bars	Develop the concept of the mean as a leveling or equalizing of data points	219	326
11.3	The Mean Foot	Develop the algorithm for finding the mean	220	326
11.4	Finding the Balance Point	Develop the concept of the mean as a balance point for all the data points	221	328

Chapter 12 Exploring Concepts of Probability

	Activity	Mathematical Goal	page number (workbook/text)	
12.1	Impossible, Possible, Certain	Explore the concepts of less-likely and more-likely events	223	340
12.2	Add Then Tally	Explore the concepts of less-likely and more-likely events	224	341
12.3	Probability Line	Explore the concepts of less-likely and more-likely events	226	341
12.4	Design a Bag	Explore the probability of an event in a one-stage experiment	227	342
12.5	Testing Bag Designs	Explore the probability of an event in a one-stage experiment	228	343
12.6	Create a Game	Explore the concept of sample space and the probability of an event in a one-stage experiment	229	345
12.7	Create a Game with Spinners	Explore the concept of sample space and the probability of an event in a one-stage experiment	231	345
12.8	Twelve Chips	Explore the probability of events in a two-stage experiment	233	347
12.9	Match	Develop the concept of sample space in a two-stage experiment	235	348
12.10	Word Problems	Explore theoretical probabilities	237	350
12.11	Checking the Theory	Explore long-run versus short-run results of a probability experiment	238	352
12.12	Experimental Probability	Develop the concept that probabilities can be estimated by long-run experiments	241	353
12.13	Word Problems	Develop the concept that probabilities can be estimated by long-run experiments	243	354

Name _____ Date _____

EXTENDING NUMBER RELATIONSHIPS TO LARGER NUMBERS:
Little Ten-Frames

Materials: Each student should have a set of 10 tens and a set of frames for each number 1 to 9 with an extra 5.

Directions: Find as many different ways to represent a given number as possible. As you write down each answer, model it with the little-ten frames.

1.	67	65 and 2 more	3 less than 70	60 and 7
		50 and 17	40 and 27	30 and 37
2.	80	_____	_____	_____
		_____	_____	_____
3.	45	_____	_____	_____
		_____	_____	_____
4.	92	_____	_____	_____
		_____	_____	_____
5.	76	_____	_____	_____
		_____	_____	_____
6.	31	_____	_____	_____
		_____	_____	_____

Extension: Break apart the numbers to perform mental addition. Use numbers to show your thought process.

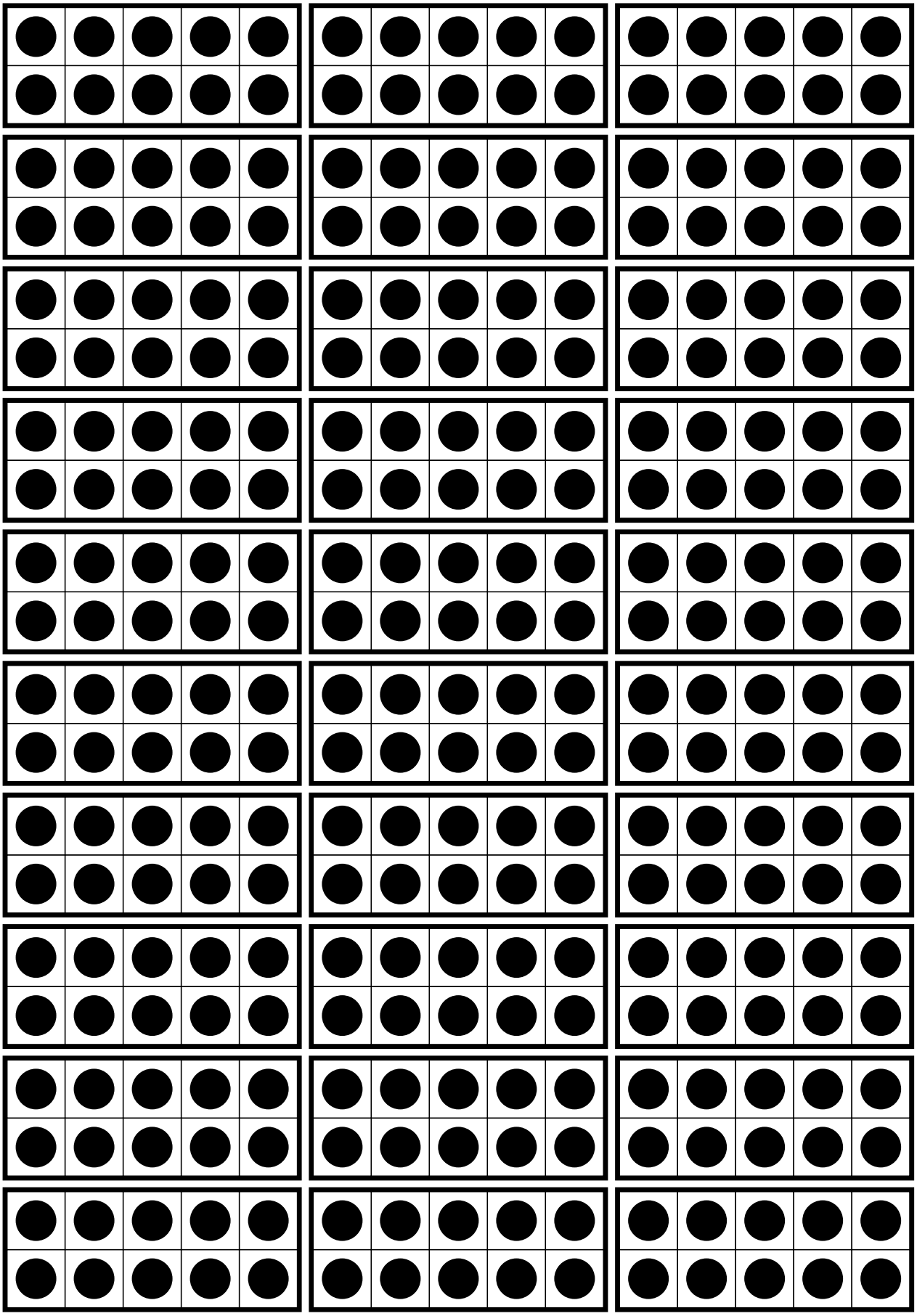
7. 67 + 56 (50 + 50) + (17 + 6) = 100 + 23 = 123

8. 33 + 45 _____

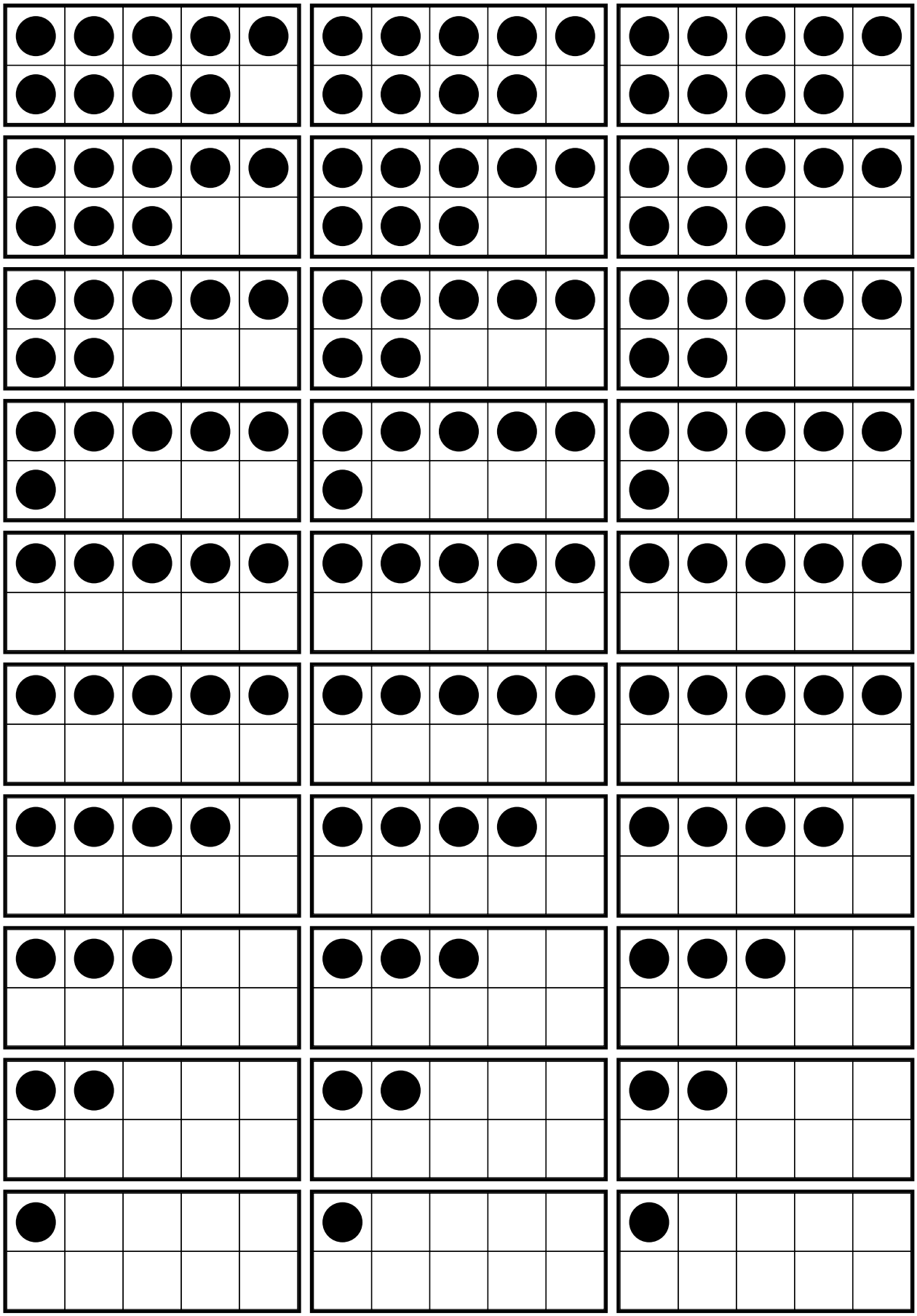
9. 48 + 29 _____

10. 15 + 73 _____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (pp. 43-44)



BLM 3—Little ten-frames

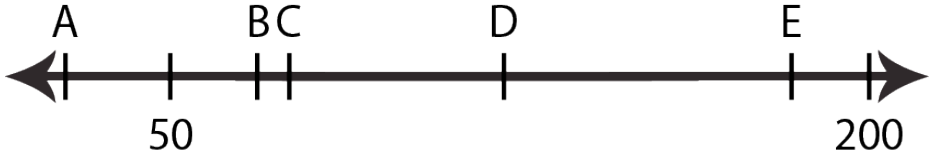


BLM 4—Little ten-frames

Name _____ 11 Date _____

RELATIVE MAGNITUDE: Who Could They Be?

What numbers do you think the different points labeled with letters might be and why do you think that?



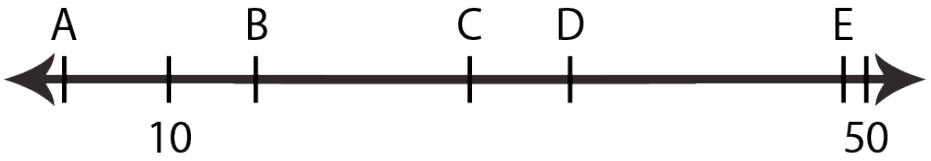
A = _____ Why do you think that? _____

B = _____ Why do you think that? _____

C = _____ Why do you think that? _____

D = _____ Why do you think that? _____

E = _____ Why do you think that? _____



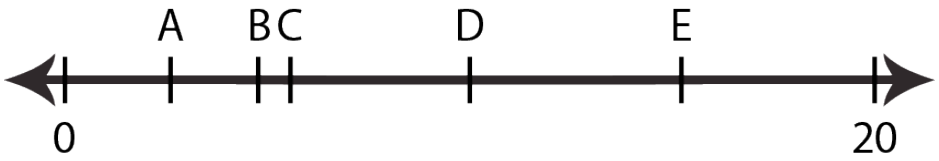
A = _____ Why do you think that? _____

B = _____ Why do you think that? _____

C = _____ Why do you think that? _____

D = _____ Why do you think that? _____

E = _____ Why do you think that? _____



A = _____ Why do you think that? _____

B = _____ Why do you think that? _____

C = _____ Why do you think that? _____

D = _____ Why do you think that? _____

E = _____ Why do you think that? _____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 45)

Directions: Work in pairs. For each number line, label 2 points with numbers (not necessarily at the ends), and 4-5 points with letters. Then, go back and work together to assign a value to each letter. After, you will present one of your number lines to the class by explaining its letters and values.

1.

2.

3.

4.

5.

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 45)

CONNECTIONS TO REAL-WORLD IDEAS: Is It Reasonable?

Directions: Select the number and the unit of things (10 kids, 20 bananas, etc.), and see what kinds of questions you can make up. After, share a set of your questions with the class.

1. Number and unit: 15 feet

1a. Could the teacher be 15 feet tall?
1b. Could your living room be 15 feet wide?
1c. Can a man jump 15 feet high?
1d. Could the school building be 15 feet tall?
1e. Could three students stretch out their arms 15 feet?

2. Number and unit: _____

2a. _____

2b. _____

2c. _____

2d. _____

2e. _____

3. Number and unit: _____

3a. _____

3b. _____

3c. _____

3d. _____

3e. _____

4. Number and unit: _____

4a. _____

4b. _____

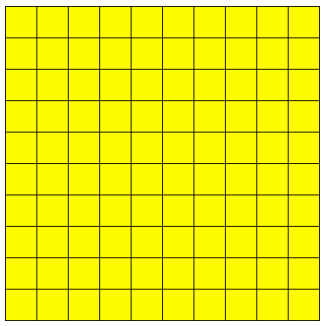
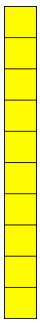
4c. _____

4d. _____

4e. _____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 46)

EXTENDING THE PLACE-VALUE SYSTEM: What Comes Next?

		
hundreds piece	tens piece	ones piece (cm)

Directions: Answer the questions below. Use the manipulatives for help.

1a. How many ones does it take to make a ten? _____

1b. How many ones does it take to make 6 tens (or 60)? _____

2a. How many tens does it take to make a hundred? _____

2b. How many tens does it take to make 7 hundreds (or 700)? _____

3a. Using #1 and #2 for help, how many hundreds does it take to make a thousand? _____

3b. Tape 10 hundreds together in order to show 1,000.

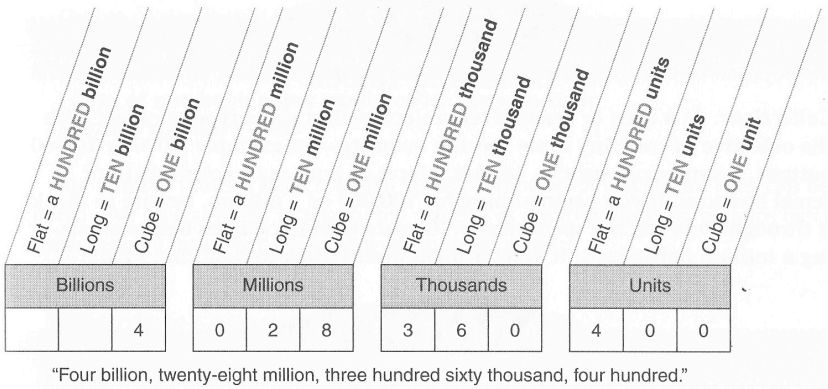
4. What is the formula for moving up a place-value? _____

5. What is the formula for moving down a place-value? _____

6. As a class, create a ten-thousand piece. Use your work from #3b as a starting point.

7. How can we extend our model to the next place-value?

Extension: Let's explore place-value in the context of three-dimensional models: *cube*, *long*, and *flat*. Explain how the place-value table follows this pattern. Use the manipulatives for help.



Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 48)

Name _____ 15 Date _____

THINKING ABOUT PARTS OF NUMBERS: The Other Part of 100

Directions: Work in pairs with a set of little ten-frame cards. One student makes a two-digit number. Then both students work mentally to determine what goes with the ten-frame amount to make 100.

Write your solutions on paper and then check by making the other part with the cards to see if the total is 100. Students take turns making the original number.

1. 64 + 36 = 100 Is your answer correct? (make the other part with the cards) Yes

2. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

3. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

4. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

5. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

6. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

7. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

8. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

9. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

10. _____ + _____ = 100 Is your answer correct? (make the other part with the cards) _____

Extension 1: Repeat the first activity, substitute 70 for 100.

11. _____ + _____ = 70 Is your answer correct? (make the other part with the cards) _____

12. _____ + _____ = 70 Is your answer correct? (make the other part with the cards) _____

13. _____ + _____ = 70 Is your answer correct? (make the other part with the cards) _____

14. _____ + _____ = 70 Is your answer correct? (make the other part with the cards) _____

Extension 2: Repeat the first activity, substitute 83 for 100.

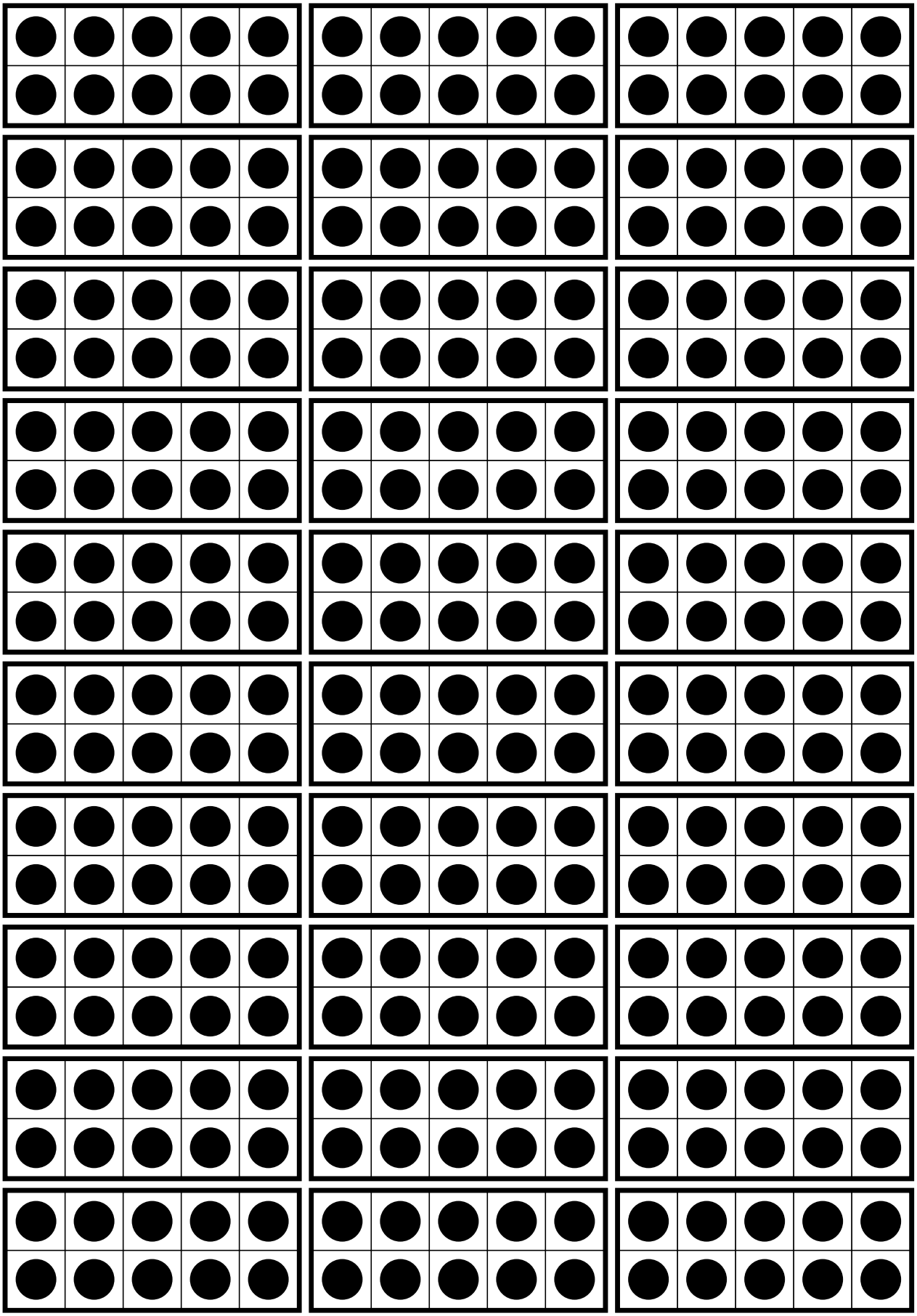
15. _____ + _____ = 83 Is your answer correct? (make the other part with the cards) _____

16. _____ + _____ = 83 Is your answer correct? (make the other part with the cards) _____

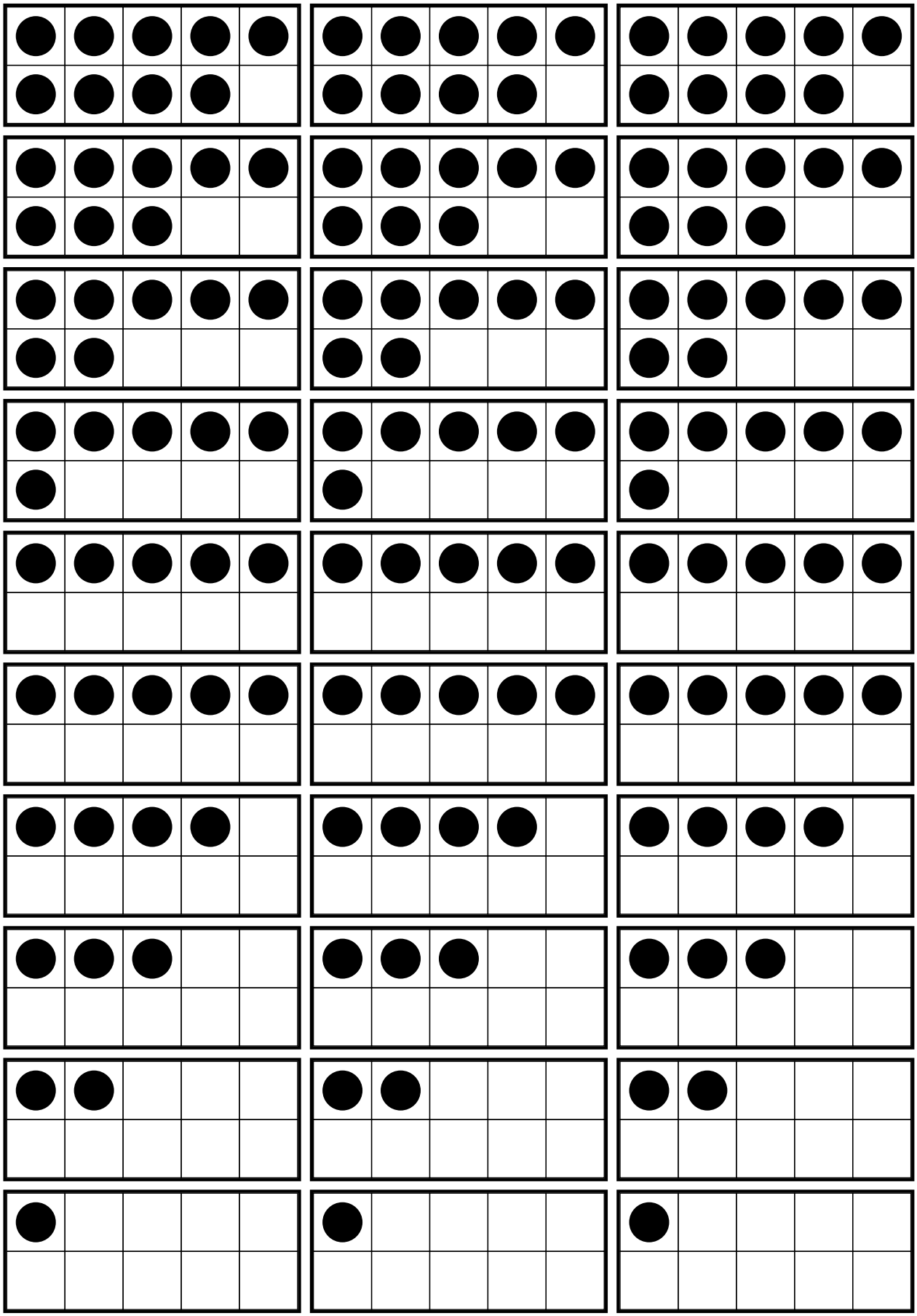
17. _____ + _____ = 83 Is your answer correct? (make the other part with the cards) _____

18. _____ + _____ = 83 Is your answer correct? (make the other part with the cards) _____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 54)



BLM 3—Little ten-frames



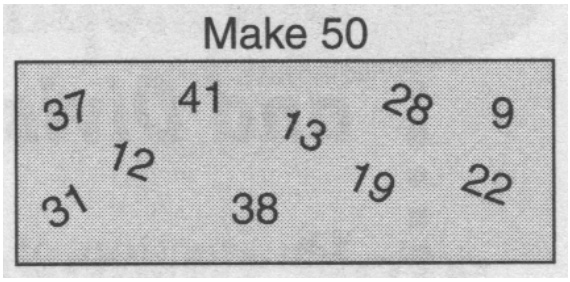
BLM 4—Little ten-frames

Name _____ 18 Date _____

THINKING ABOUT PARTS OF NUMBERS: Compatible Pairs

"Compatible numbers for addition and subtraction are numbers that go together easily to make nice numbers. Numbers that make tens or hundreds are the most common example. Compatible sums also include numbers that end in 5, 25, 50, or 75, since these numbers are easy to work with as well."

Directions: Connect the compatible pairs to make the designated sum.



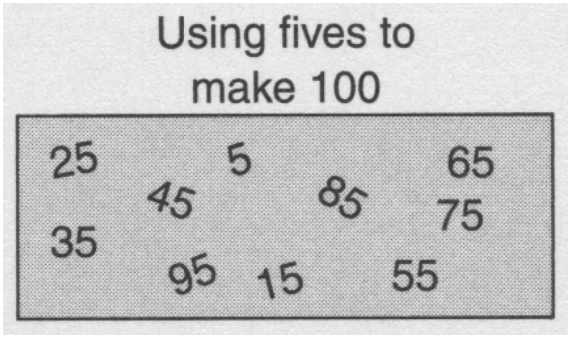
1. _____ + _____ = 50

2. _____ + _____ = 50

3. _____ + _____ = 50

4. _____ + _____ = 50

5. _____ + _____ = 50



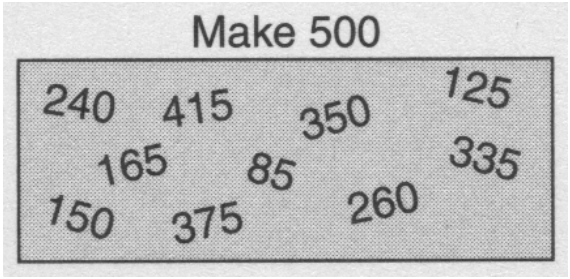
6. _____ + _____ = 100

7. _____ + _____ = 100

8. _____ + _____ = 100

9. _____ + _____ = 100

10. _____ + _____ = 100

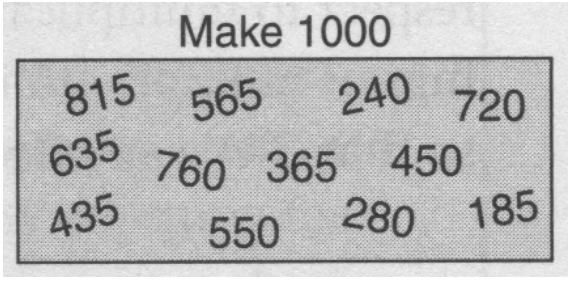


11. _____ + _____ = 500

12. _____ + _____ = 500

13. _____ + _____ = 500

14. _____ + _____ = 500



15. _____ + _____ = 1,000

16. _____ + _____ = 1,000

17. _____ + _____ = 1,000

18. _____ + _____ = 1,000

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 55)

THINKING ABOUT PARTS OF NUMBERS: Calculator Challenge Counting

Directions: Press any number on the calculator (e.g., 17), then + 8. Say the sum before you press the =. Then continue to add 8 mentally, challenging yourself to say the number before you press the =. See how far you can go before making a mistake.

The constant addend in "Calculator Challenge Counting" can be any number, even a two- or three-digit number. As an added challenge, reverse the process by pressing – followed by the same number and then =, =, …

After, discuss patterns that appear.

Name _____ 20 Date _____

THINKING ABOUT PARTS OF NUMBERS: Little Ten-Frame Addition and Subtraction

Directions: Working in pairs, each student has a set of little ten-frame cards. Each student makes a number with his or her cards. When both have their numbers ready, they place it out so both can see. Then they try to be the first to tell the total.

For the subtraction version, one student makes a number greater than 50 and the other writes a number on paper that is less than 50. The written number is to be subtracted from the modeled number.

Share strategies to see how fast you can get the answers.

1. When adding, what are the best strategies that you used to get your answers?

_____

_____

_____

_____

_____

_____

_____

_____

2. When subtracting, what are the best strategies that you used to get your answers?

_____

_____

_____

_____

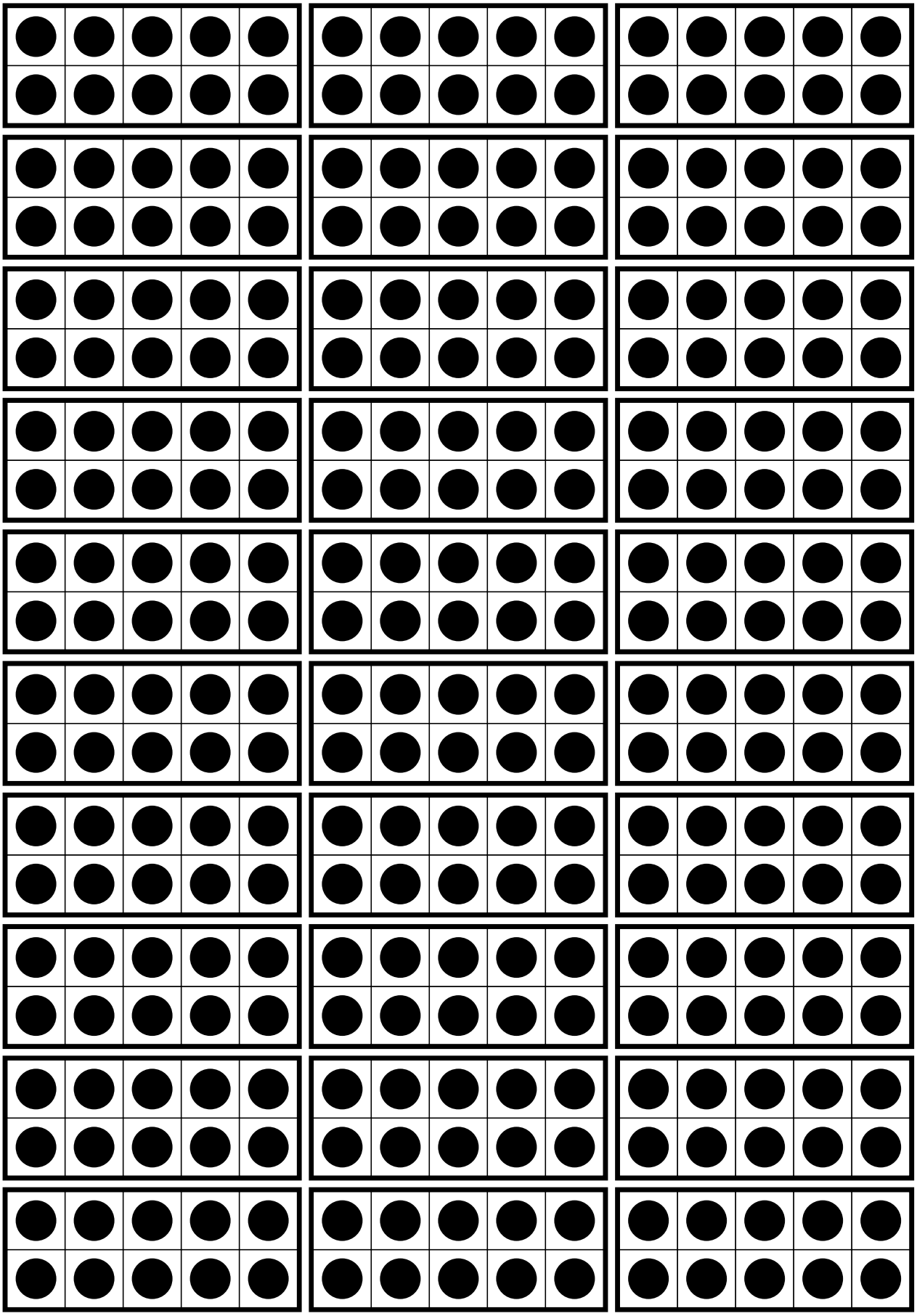
_____

_____

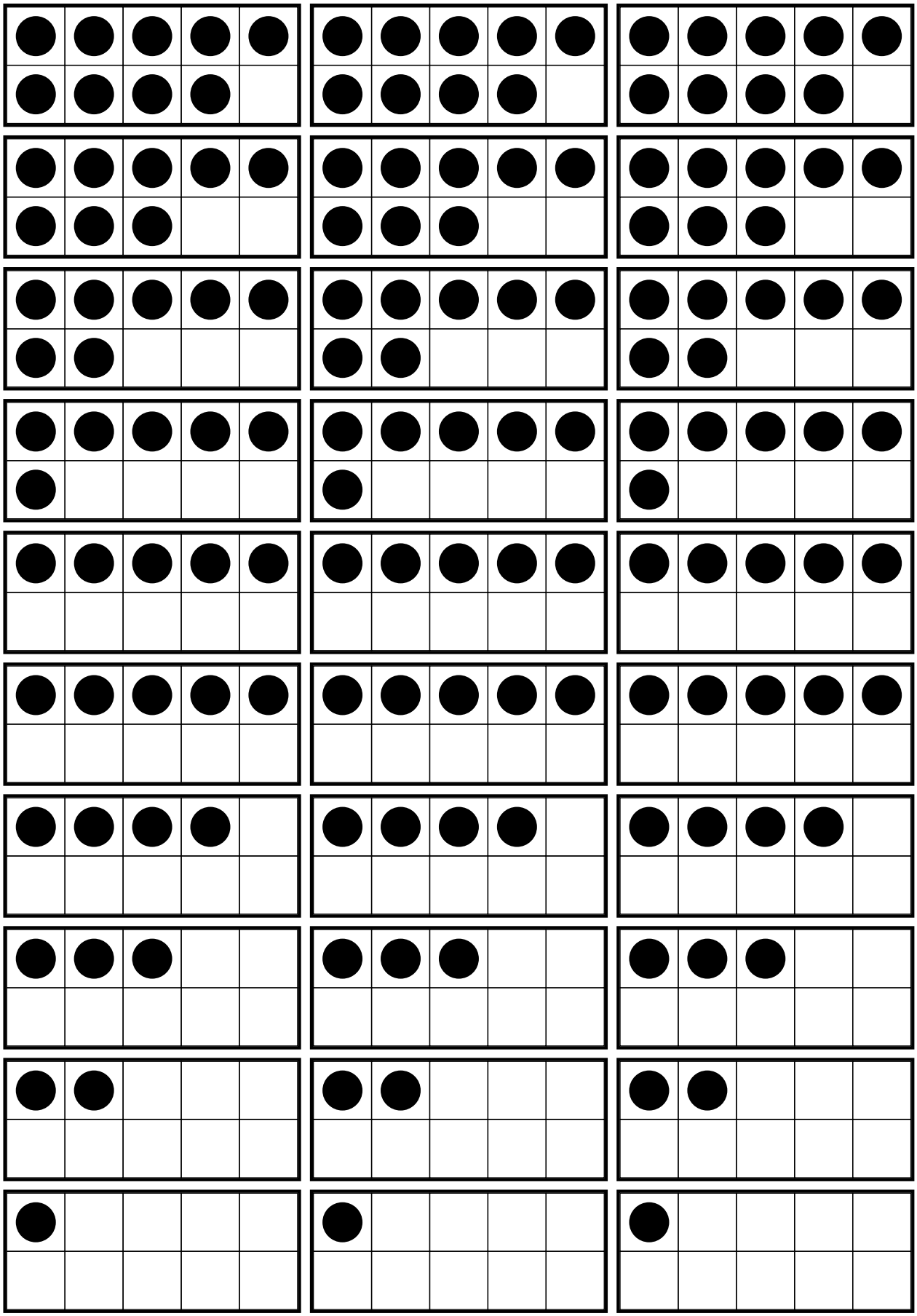
_____

_____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (p. 55)



BLM 3—Little ten-frames



BLM 4—Little ten-frames

Name _____ 23 Date _____

REMAINDERS: Interpret the Remainder

In real contexts, remainders sometimes have three additional effects on answers:

- **a.** The remainder is discarded, leaving a smaller whole-number answer
- **b.** The remainder can "force" the answer to the next highest whole number
- **c.** The remainder becomes the answer

Directions: Solve using manipulatives. Write *a*, *b*, or *c* to explain how you interpret the remainder.

1. Each can holds 9 people. There are 82 people traveling. How many vans will be completely full?

9, a

2. You have 30 pieces of candy to share fairly with 7 children. How many pieces of candy will each child receive?

_____

3. The ferry can hold 8 cars. How many trips will it have to make to carry 25 cars across the river?

_____

4. Each car in the amusement park ride can hold 6 people. There are 34 people waiting in line. How many people will be on a car that is not full?

_____

5. The rope is 25 feet long. How many 7-foot jump ropes can be made?

_____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (pp. 61-62)

Extension: Write six word problems, two for each of the three ways in which a remainder can be interpreted. (Make sure to mix up the order of your problems.) After, exchange problems with a partner. Your partner should solve the problems using manipulatives, and write *a*, *b*, or *c* to explain how the remainder is interpreted.

6. _____

_____

_____

7. _____

_____

_____

8. _____

_____

_____

9. _____

_____

_____

10. _____

_____

_____

11. _____

_____

_____

12. _____

_____

_____

Adapted from *Teaching Student-Centered Mathematics: Grades 3-5* (pp. 61-62)